

# The *ix*-diagram for air + water vapor mixtures<sup>1</sup>

by Richard Mollier, Dresden

This article extends the application of the earlier *ix*-diagram by the author in particular to mixtures of air, water vapor, water and ice. It discusses the processes occurring when moist air contacts a water or icy surface. It also derives the theory of the psychrometer and the Lewis' law with the help of the diagram.

In 1923 I published a graphical representation<sup>2</sup> that significantly facilitates the solution of many problems arising when considering mixtures of air and water vapor. The diagram has since then demonstrated its value, has been extended in many ways, and is in ever wider use. Today, I would like to discuss some extensions of the process, and summarize more exactly its foundations.

The diagram is valid for changes of state that take place at constant total pressure. Its applications are not limited to mixtures of air and water vapor, but may as well be advantageously applied to mixtures of other gases and vapors, e.g. to mixtures of air and combustible vapors. In the following only mixtures of water vapor and air are considered. In most applications considered, the partial pressures are small, thus permitting to consider both substances as ideal gases, and when not particularly high temperatures are considered, the specific thermal capacities of both substances may be considered constant as well.

## Notation and main relationships

- $p$  partial pressure of water vapor
- $p'$  saturation pressure of water vapor at the temperature of the mixture
- $p_0$  total pressure (barometric pressure)
- $x$  the abscissa in the diagram, is the mass ratio of water vapor to air in kg/kg, i.e. the mass of water vapor in one kg of air. It is in general:  $x = \frac{M_W}{M_A} \frac{p}{p_0 - p'}$ , where  $M_W$  and  $M_A$  are the molar masses of water and air, respectively. The dimensions of  $x$  as kg/kg are as usual. It would be simpler if this magnitude were expressed in mol/mol, then it would simply be:  $x = \frac{p}{p_0 - p'}$ .
- For water vapor - air mixtures it becomes:  $x = 0.622 \frac{p}{p_0 - p'}$
- $x'$  Maximum value of the water content in the air, for  $p = p'$ ,  $x' = 0.622 \frac{p'}{p_0 - p'}$
- $m_{DA}$  the mass of air (dry) in a mass of mixture, in kg. The mass of the mixture is then:  $m_{DA} (1 + x)$

---

<sup>1</sup> Translation of the original as included in "Festschrift Prof. Aurel Stodola zum 70. Geburtstag", published by E. Honegger, Orel Fuessli Verlag, Zurich and Leipzig, 1929, pp. 438-452. Later published in Zeitschrift des Vereins deutscher Ingenieure, 73(29), 1009-1013.

This translation has been done by Dr Manuel Conde-Petit. Comments and a review of the translation by Mr Donald P. Gatley, PE, author of 'Understanding Psychrometrics', are gratefully acknowledged. The translation is offered *in memoriam* of Professor Richard Mollier on the 70<sup>th</sup> anniversary of his death.

<sup>2</sup> See the References at the end of the article.

- $C_{DA}$  Specific thermal capacity of dry air at constant pressure. In the following I shall take  $C_{DA} = 0.24^3$ .
- $C_W$  Specific thermal capacity of the water vapor at constant pressure. In the following I shall take  $C_W = 0.46$ .
- $i$  the ordinate in our diagram, is the thermal energy content (enthalpy) for 1 kg air +  $x$  kg water vapor:  $i = 0.24 t + x (0.46 t + 595)$ . 595 is the vaporization enthalpy of water at 0 °C.
- $i'$  the enthalpy of air saturated with water vapor:  $i' = 0.24 t + x' (0.46 t + 595)$ .  $i'$  and  $x'$ , at a given total pressure, depend only upon the temperature.
- $\varphi$  relative humidity
- $\psi$  degree of saturation
- $r$  enthalpy of vaporization

### The $ix$ -diagram (Fig. 1)

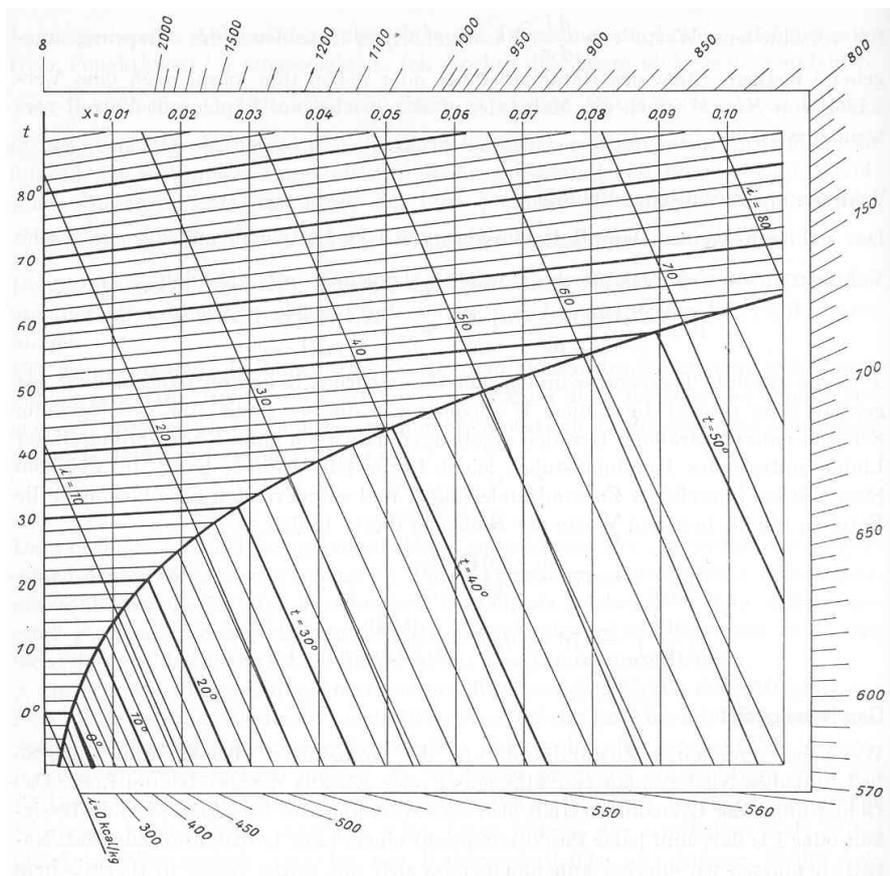


Fig. 1 - The vertical ordinate axis,  $x = 0$ , represents the states of dry air. Its point at 0 °C is the origin. The abscissae axis goes through this point, and the marks of the edge scale. The *boundary curve* represents saturated states at 1 atm, and separates the unsaturated from the saturated regions. The isotherms are drafted at 2 °C intervals.

<sup>3</sup> NT: The original units of the paper are kept: kcal for energy and  $\text{kg}/\text{m}^2$  for pressure.

Oblique coordinates are advantageous for the  $ix$ -diagram. They permit a better use of the graph area and the most important families of curves are clearly represented. Following the above formula for the enthalpy, the isotherms are straight lines. It is particularly convenient, for a vertical enthalpy axis, to choose the angle of the coordinates such that the  $0\text{ }^\circ\text{C}$  isotherm is horizontal. In a certain sense one has, in addition to the main diagram, a second orthogonal system with  $x$  as abscissa and  $i_0 = t(0.24 + 0.46x)$ , i.e. the enthalpy referred to water vapor at  $0\text{ }^\circ\text{C}$ , as ordinate, which is very convenient to draw the diagram. Of course, any other angle can as well be selected for the coordinates.

*The boundary curve*

The isotherms are independent of the choice of total pressure. This shows up when drawing the saturation line, the *boundary curve*. The *boundary curve* has  $i'$  and  $x'$  as coordinates, and is drafted by finding the point with abscissa  $x'$  on the isotherm. In general, the *boundary curve* is drafted for an average barometric pressure. Sometimes it may be necessary to consider rather low pressures, as for example in vacuum drying. Below  $0\text{ }^\circ\text{C}$ , the *boundary curve* for thermal equilibrium over ice shall be represented. In Fig. 2, I draw as well the *boundary curve* for equilibrium over subcooled water (dotted line).

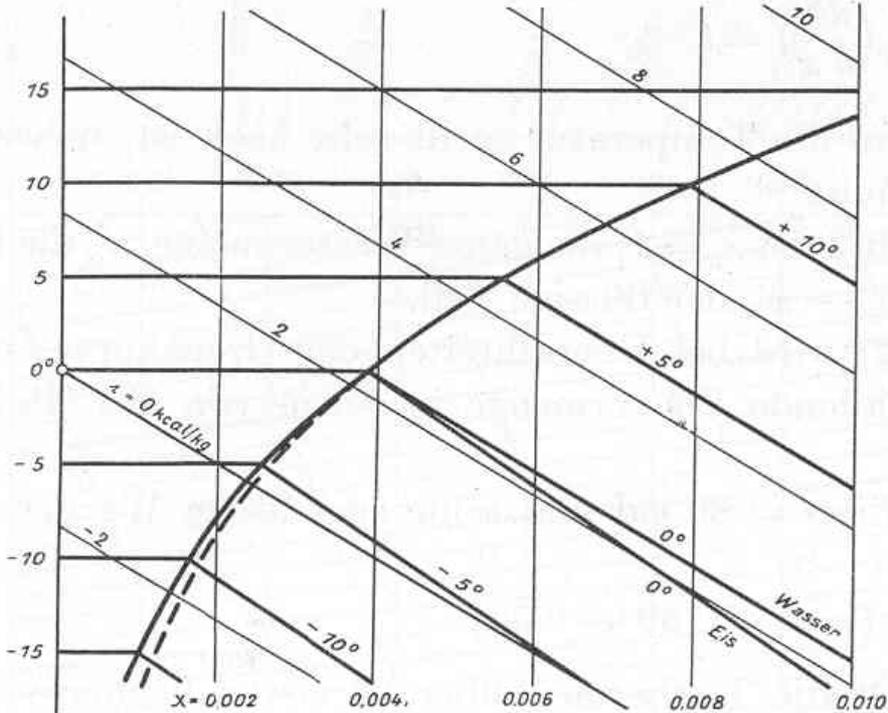


Fig. 2 -  $ix$ -diagram for the freezing point region.

In my first paper on the diagram, I have also shown the lines of “constant relative humidity”,  $\varphi$ . I would not recommend this any more.  $\varphi = \frac{p}{p'}$  represents the ratio of the water vapor mass available in a given space to maximum possible mass of water vapor in that space. It is simpler, and better adapted to our methods of calculation, when we use the degree of saturation of the air

$\psi^4$ , which is defined by the ratio  $\frac{x}{x'}$ . Lines of constant degree of saturation are constructed by simply dividing the isotherms between the ordinate axis and the *boundary curve* into equal parts and join the corresponding points. For temperatures higher than that corresponding to saturation at the current total pressure  $\psi = \infty$ , i.e. the air will accept any amount of water vapour. The  $\psi$  lines are asymptotic to the corresponding isotherm. The following relationship exists between  $\varphi$  and  $\psi$ :  $\frac{\psi}{\varphi} = \frac{p_0 - p'}{p_0 - p}$ .

It follows from here that, at normal ambient air temperatures, the difference between the two magnitudes is minimal. Hence, the practical experience with the relative humidity  $\varphi$  in the meteorology may also apply to the degree of saturation  $\psi$ . Ascertaining the degree of saturation in the  $ix$ -diagram is so simple that drawing  $\psi$ -lines in the diagram is not even necessary. On the other hand,  $\varphi$  and  $\psi$  give only limited information on the degree of saturation of the air. They permit only to recognize how much water the air is able to take when the temperature remains constant, although an important amount of energy is required to that effect. It is however practical, and often important, to know the degree of saturation when saturating the air without addition of energy (adiabatically). It is then necessary to compare  $x$  with  $x'$  at equal enthalpy or at the *cooling boundary* (more on this later).

### The edge scale

In many tasks it is desirable to define the direction of a change of state of the air  $\frac{di}{dx}$ , or to rapidly draw the line linking two states  $\frac{i_2 - i_1}{x_2 - x_1}$  in the diagram. This is facilitated by an edge scale (Fig. 1) that defines lines through the origin for various values of  $\frac{di}{dx}$ . Instead of the edge scale, or together with it, one may advantageously use a scale on transparent paper, as suggested by Hirsch<sup>5</sup>.

### Volume and specific volume

The volume  $V$  of an air water vapor mixture with a 1 kg air content, and the specific volume  $v$  as well as the density  $\frac{1}{v}$  are calculated as follows:

$$V = \frac{47.1 (0.622 + x) T}{p_0} \quad v = \frac{47.1 (0.622 + x) T}{p_0 (1 + x)}$$

<sup>4</sup> The magnitude  $\psi$  has already been used by *Zeuner*. See Technische Thermodynamik, pp. 321, 1890. NT: *Zeuner, Gustav*, Technische Themodynamik, Vol. 2, Arthur Felix Verlag, Leipzig, 1890, pages 309 and 323. Zeuner refers to the ratio of humidity ratios as relative humidity.

<sup>5</sup> See Reference 10.

$T$  is the absolute temperature and  $p_0$  is the total pressure in  $\text{kg}_f/\text{m}^2$ . Grubenmann<sup>6</sup> showed that the  $V=\text{Const.}$ , as well as the  $v=\text{Const}$  lines result in families of practically parallel lines. It is thus very simple to determine, with the edge scale, the variations of  $V$  and  $v$  due to changes of state, without drawing those lines in the diagram. I shall give here the formulae for the exact values of the directions of those lines:

$$\left(\frac{di}{dx}\right)_v = 470 + \frac{0.046 T}{0.622 + x}$$

$$\left(\frac{di}{dx}\right)_v = 470 + \left(0.46 + \frac{0.046}{0.622 + x} - \frac{0.22}{1 + x}\right) T$$

## The fog region

Whenever the *boundary curve* is crossed by an air change of state, it means that precipitation of either liquid water or ice from the saturated air takes place. The region below the *boundary curve* represents then a mixture of gas, vapor and liquid or ice, and each point represents a very well defined state. One has, of course, to assume that the whole mass is in thermal equilibrium. This is possible in particular when the liquid water fluctuates in the air as extremely fine droplets (fog). It shall now be necessary to extend the isotherms past the *boundary curve*, and they will naturally have a different slope in the fog region.

$x$  is now the total mass of the second substance that is mixed in the mass unit of gas. From this,  $x'$  is amount in vapor form according to the temperature of the mixture, and the remainder  $x - x'$  is liquid or solid.

The enthalpy of the whole mixture is, for air water vapor mixtures:  $i = i' + (x - x')t$ . The isotherms in the fog region are also straight lines, since  $x'$  and  $i'$  depend only upon the temperature, and their slope is given by:  $\left(\frac{di}{dx}\right)_t = t$  (Fig. 1). They are almost parallel to the isenthalps ( $i = \text{Const}$ ) at moderate temperatures.

From the total amount of water  $x_3 = x_1$  at point 3 in the fog region, (Fig. 3),  $x'_3$  is in the vapor phase while  $x_1 - x'_3$  is in the liquid phase. When crossing the *boundary curve* at temperatures below zero ( $0^\circ\text{C}$ ), the amount of precipitated water, in the case of thermal equilibrium, shows up as ice (frost, snow, icy fog). Considering the fusion enthalpy of ice ( $= 80 \text{ cal}$ ) and its specific thermal capacity ( $= 0,5$ ), we get:

$$i = i' - (x - x')(80 - 0.5t) \quad \text{and} \quad \left(\frac{di}{dx}\right)_t = -80 + 0.5t .$$

While above zero ( $0^\circ\text{C}$ ) the (fog region) isotherms are less steep than the  $i$ -lines, they are steeper below zero. At zero ( $0^\circ\text{C}$ ) there are two isotherms: a liquid water and an ice isotherm that cross at the *boundary curve*. The region between these two isotherms corresponds to a mixture of air, water vapor and liquid and solid water. The ratio of liquid and solid water amounts defines a particular line in this region. In case the  $ix$ -diagram is drafted for a single total pressure, it is simple to draw the isotherms in the fog region. It is however desirable to distinguish them from

<sup>6</sup> See Reference 3.



When two saturated amounts of air mix, it always results in a foggy state. The amount of precipitation may readily be obtained from the diagram. If we have a state 1 in the fog region and unsaturated air in state 2, it is easy to calculate the amount of state 2 air to be mixed with the foggy air in order to dissipate the fog. We join the two points of state with a straight line. Its intersection with the *boundary curve* gives the maximum possible value of  $x_m$ , and the mixing ratio is

$$\frac{m_{DA,1}}{m_{DA,2}} = \frac{x_m - x_2}{x_1 - x_m}.$$

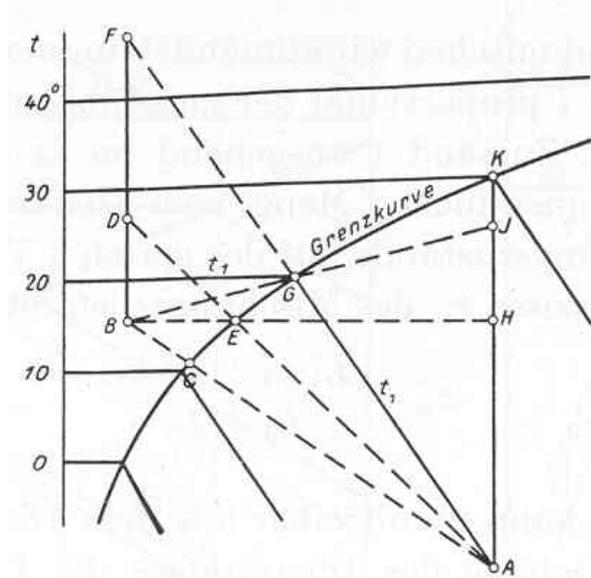


Fig. 4 - Fog dissipation through mixing of unsaturated air and through heating.

enthalpy  $\frac{Q}{m_{DA,2}} = \overline{BD}$  the air attains the state  $D$ , and the required mixing ratio will now be

$$\frac{m_{DA,2}}{m_{DA,1}} = \frac{\overline{AE}}{\overline{DE}}.$$

In the case that by the fog dissipation the initial temperature  $t_1$  of the air should not change, we would have to warm-up the unsaturated air by  $\frac{Q}{m_{DA,2}} = \overline{BF}$  up to point  $F$ , in order for

the end state  $G$  to be attained. The enthalpies  $\overline{BD}$  and  $\overline{BF}$  refer to 1 kg of the starting state 2 (point  $B$ ). For comparison we shall now refer the enthalpies to 1 kg of the foggy air. By drawing the lines  $BH$  and  $BJ$ , through  $E$  and  $G$ , respectively, we obtain:  $\frac{Q}{m_{DA,1}} = \overline{AH}$  and  $\frac{Q}{m_{DA,1}} = \overline{AJ}$ , respectively.

We may dissipate the fog without adding extra air by simply supplying the enthalpy  $\frac{Q}{m_{DA,1}} = \overline{AK}$ .

We should also note that the enthalpy  $\overline{AK}$  is not always necessarily larger than that required by mixing unsaturated air. This may be recognized if we think of  $B$  at a lower temperature.

### Adding water vapor, or liquid water, to air

When we add water vapor or liquid water to air at state  $I$ , its state changes again linearly in the  $ix$ -diagram, since both  $x$  and  $i$  increase proportionally to the added amount. The slope of the straight lines is given by  $\frac{di}{dx} = i_0$ , where  $i_0$  is the enthalpy of 1 kg of the added water vapor or

If the mixing of two amounts of air concurs with the supply, or loss, of the energy  $Q$ , we may determine the end state by vertically adding, or subtracting the enthalpies  $\frac{Q}{m_{DA,1}}$

or  $\frac{Q}{m_{DA,2}}$ , respectively, and proceed with the

mixing calculations as before. Fig. 4 shows an arbitrary foggy state  $I$ , point  $A$ , and unsaturated air at state 2, point  $B$ , and how, by mixing both, the fog may be lead to dissipate. To attain this, the end point of state will have to lie on the *boundary curve*. Wouldn't we heat the air, the required minimum mixing

ratio would be  $\frac{m_{DA,2}}{m_{DA,1}} = \frac{\overline{AC}}{\overline{BC}}$ . If we would warm-

up the air before mixing, the required amount to mix would be reduced. By supplying the

liquid water. In this last case is  $i_0 = t_0$ . The diagram shows that precipitation always takes place, when departing from air states near ambient temperatures, and considering the edge scale, we gradually add saturated water vapor. If the enthalpy of the water vapor is larger than  $640 \text{ cal}$ , at  $p_0 = 1 \text{ atm}$ , the air state will leave the fog region with further supply of vapor. Otherwise it will not leave that region.  $x'_1 - x_1$  and  $x'_2 - x_2$  (Fig. 5) show the maximum amount of water vapor that may be added to one kg of air before it attains the fog region. Or, if  $G_0$  kg of water vapor is supplied to a space per unit of time, the amount  $m_{DA} = \frac{G_0}{x' - x_1}$  must be blown through the space in order to avoid fog formation. This amount may be reduced by warming-up the air before supply, since  $x'$  is increased in the process (Fig. 5).

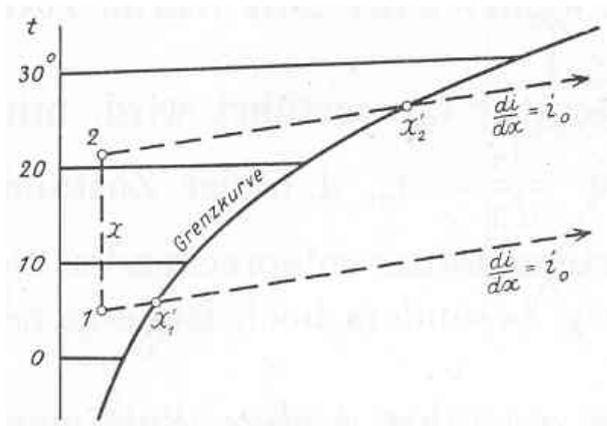


Fig. 5 - Fog creation through vapor supply to a space, and avoiding fog through ventilation and heating.

If we add liquid water at the temperature  $t_0$  to the air, with which it comes in thermal equilibrium,  $Q = 0$  and  $\frac{di}{dx} = t_0$ , i.e. the state of the air approaches a fog region isotherm equal to the water temperature, or, when the water temperature is not particularly high, moves along a line of  $i = \text{Const}$ .

Instead of water we may as well mix ice with the air (sublimation). The change of state approaches an icy fog isotherm corresponding to the ice temperature.

One may conclude from the last paragraphs what means may be used to attain a desired state of the air, for example in heating and ventilation plants, and how advantageous it is to use the  $ix$ -diagram to this endeavour.

### Air at given initial state flows over a liquid water or ice surface at given temperature

A thin boundary layer of air always lies above a liquid water or ice surface, which is saturated with water vapor and is at the temperature of the water or ice. When a mass of air at a given state comes into contact with that surface, a mixing process will take place between the air and the boundary layer. Through this process, following our explanation above, the state of the air experiences a change that, in the  $ix$ -diagram, follows the line joining the original state of the air with a point at the *boundary curve* corresponding to the temperature of the water or ice. If the original water content of the air  $x$  is smaller than that of the boundary layer  $x'$ , the air will take water vapor from the water (evaporation). If it is larger, water vapor will precipitate from the air into the water (condensation).

The criterion  $(x > x') \vee (x < x')$  coincides with that of Dalton  $(p > p') \vee (p < p')$ . The mixing of the air with the boundary layer takes place gradually. At first a small amount of air mixes with the boundary layer, and after that the mixed boundary mixes again with a fresh new amount of air, and so on. The state of all these mixed boundaries have to lie on the above defined straight line. This is valid as long as the water temperature remains constant. Hence, this defines only the initial direction of the change of state of the air, due to its contact with the water. If we assume

that no heat transfer with the environment takes place, the water temperature can only remain constant if it corresponds to the fog isotherm that passes through the initial state of the air. In that case, the energy needed to evaporate and warm-up the vapor is supplied by the air, that is cooled down in the process. We have then exactly the case discussed in the last section, where we thought that water at a given temperature, added to an air stream at a given initial state, will come into equilibrium with the air, without an external supply of energy. If the water is at a temperature higher than that mentioned, the mixing process with the boundary layer will require external energy. That energy will be taken from the water. The water will be cooled down until it reaches steady state at that limiting fog isotherm value.

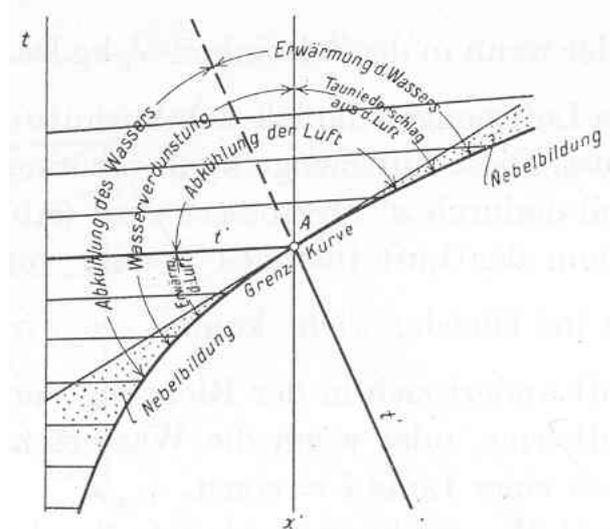


Fig. 6 - Interactions between air and a wet or icy surface in contact with it:  
*Grenz-Kurve* - Boundary curve;  
*Nebelbildung* - Fog formation;  
*Abkühlung des Wassers*- Water cooling;  
*Wasserverdunstung* - Water evaporation;  
*Erwärmung der Luft* - Air heating;  
*Abkühlung der Luft* - Air cooling;  
*Tauniederschlag aus der Luft* - Dew formation  
*Erwärmung des Wassers* - Water heating.

position of the air state point in relation to the following five lines: the boundary curve, the tangent to it at point *A*, the line  $x = \text{const.}$ , the isotherm through *A*, and the prolongation of the fog region isotherm through *A*.

## The Psychrometer

The process in the Psychrometer of August<sup>7</sup>, which is now hundred years old, corresponds exactly to the process described in the above section. If we may assume that no heat transfer takes place with the environment, the wet bulb thermometer shows the cooling limit. The deviations due to heat transfer with the environment in aspiration psychrometers, may be reduced through

When the initial water temperature is lower than the limiting value, the mixing process frees energy that will be taken by the water, as long as its temperature is lower than the limiting one. Each of the limiting water temperatures associated with a given air state is called in the practice *the cooling limit*.

It may be determined in the *ix*-diagram by simply extending the family of fog region isotherms into the unsaturated region. The air states that lie on the prolongation of those isotherms have the same cooling limit.

Everything said above for an amount of air contacting a surface of liquid water is valid as well for an icy surface and the boundary layer above it.

Point *A* in Fig. 6 represents the state of air in the boundary layer above a free surface of water, or ice, at the temperature  $t'$ . When unsaturated air, or in the limiting case just saturated air, comes into contact with this surface, the direction of the processes taking place will be determined by the relative

<sup>7</sup> NT: Ernst Ferdinand August, Ueber die Verdunstungskälte und deren Anwendung auf Hygrometrie, Poggendorfs Annalen der Physik und Chemie, 5(1825), Part I: 69-88, Part II: 335-344.

intensification of the internal heat transfer in relation to the unavoidable external one, and through good insulation.

The state of the air to be determined, may be readily obtained from the  $ix$ -diagram, under the pre-requisite that the wet bulb thermometer shows the cooling limit  $t'$ . It is only necessary to extend the fog region isotherm  $t'$  until it crosses the air state isotherm  $t$  (dry bulb thermometer).

Numerically  $x$  is calculated from the equation:  $\frac{i'-i}{x'-x} = t'$  and from the equation given before for

$i$  and  $i'$  we get:  $x = x' - \frac{C_{DA} + x' C_W}{r' + C_W (t - t')}$ .

In this equation, which is not specific to air water vapor mixtures,  $x'$  relates to the temperature of the wet bulb thermometer, and  $x$  to the state of the air being measured.  $r'$  is the vaporization enthalpy of the water at the temperature  $t'$ . For air water vapor mixtures, one obtains, with  $r' = 595 - 0.54 t'$

$$x' - x = \frac{0.24 + 0.46 x'}{595 + 0.46 t - t'} (t - t') = \frac{C'}{Q} (t - t')$$

here,  $C'$  is the specific thermal capacity of the moist air with the water vapor content  $x'$ ,  $Q$  is the energy required to vaporize  $1 \text{ kg}$  of water at the temperature  $t'$  and to warm it up to the temperature  $t$ .

The following simple formula is used to evaluate measurements in the meteorology:

$p' - p = K p_0 (t - t')$ , where  $p'$  is the saturation pressure of water vapor at the temperature  $t'$ , and  $p$  is the partial pressure of the water vapor in the air.  $p_0$  is the total pressure (barometer pressure).  $K$  is the so-called psychrometer constant, which usually takes the value  $0.00066$  for an aspiration Psychrometer. The limiting value of  $K$ , in the case that the wet bulb thermometer shows exactly the cooling limit, may be easily calculated from the equation above for  $x' - x$  if we replace  $x'$  and  $x$  by  $p'$  and  $p$ , respectively. Let's further consider  $\overline{C_{DA}}$  and  $\overline{C_W}$  the specific thermal capacities referred to one mol. We obtain then for the limiting value of  $K$ , for any gas-vapor mixture, the expression:

$$K = \frac{\overline{C_{DA}} - (2 \overline{C_{DA}} - \overline{C_W}) \frac{p'}{p_0} - (\overline{C_W} - \overline{C_{DA}}) \left( \frac{p'}{p_0} \right)^2}{M_W r' + (\overline{C_W} - \overline{C_{DA}}) \left( 1 - \frac{p'}{p_0} \right) (t - t')}$$

It results from here that, if we use the mean value of the specific thermal capacity of air, between  $0^\circ \text{C}$  and  $20^\circ \text{C}$ , given by the 'Physikalisch-technischen Reichsanstalt' as  $0.2407$ , and maintain  $C_W = 0.46$ ,

$$K = \frac{0.388 - 0.314 \frac{p'}{p_0} - 0.074 \left( \frac{p'}{p_0} \right)^2}{r' + 0.074 \left( 1 - \frac{p'}{p_0} \right) (t - t')}$$

where  $r' = 595 - 0.54 t'$ . The second order term in the numerator is extraordinarily small and may be ignored. The second term in the denominator is also less than 2% of the vaporization enthalpy. The influence of the barometric pressure is equally small in most cases, so that it may be replaced by an average value, say 750 mm Hg. Hence we obtain:

$$K = \frac{0.388 - 0.00042 p'}{595 - 0.54 t'}$$

In this equation  $p'$  shall be used in mm Hg. The following table shows the remaining influence of  $p'$ :

$t' \text{ }^\circ\text{C}$	=	0	5	10	15	20	25	30	35	40	45	50
$10^6 K$	=	649	651	652	652	651	650	647	643	636	627	615

$K$  goes through a flat maximum that lies in the most used temperature range. The average value in that range  $K = 0.00065$  agrees quite well with experimental values for the aspiration Psychrometer. One may thus conclude that, in this instrument, the wet bulb thermometer shows very closely the cooling limit.

For  $t' < 0 \text{ }^\circ\text{C}$  the wick of the wet bulb will be frozen, instead of wet. We shall change our equations accordingly, taking into account the lower enthalpy of ice, which, when referred to water at  $0 \text{ }^\circ\text{C}$ , is  $-80 + 0.5 t'$ . We get

$$x' - x = \frac{0.24 + 0.46 x'}{675 + 0.46 t - 0.5 t'} (t - t')$$

and the formula for the Psychrometer constant becomes

$$K = \frac{0.338 - 0.00042 p'}{675 - 0.5 t'}$$

from where it follows:

$t' \text{ }^\circ\text{C}$	=	-40	-30	-20	-10	0
$10^6 K$	=	593	589	584	580	578

These values agree also very well with empirically determined constants, which are given as 0,00058 for strongly ventilated Psychrometers.

## The Lewis' Law<sup>8</sup>

Dalton established the law describing the process in which water evaporates into the air, when an air stream contacts a free surface of water, or water vapor condensates into the water through dew formation. Per unit of surface area, it says:

$$\frac{dW}{dz} = \delta \frac{p' - p}{p_0}$$

where the evaporation number  $\delta$  depends in particular from the state of agitation of the air (convection). *Lewis* stated above all that instead of the pressure one could as well use the water content of the air  $x$  and that of the boundary layer  $x'$  with the result

$$\frac{dW}{dz} = \sigma (x' - x)$$

At moderate temperatures, for which we may set  $\frac{p}{p_0 - p} \approx \frac{p}{p_0}$ , both formulae give identical results. The Lewis' formula adapts itself very well to the usual methods of calculation and even better to the  $ix$ -diagram.

Furthermore, *Lewis* showed that the evaporation number  $\sigma$  relates rather simply to the heat transfer coefficient in the expression for heat transfer between the air and the water vapor:

$$\frac{dQ}{dz} = \alpha (t - t')$$

It is namely,  $\sigma C' = \alpha$ , where  $C'$  is the specific thermal capacity of the air with the water content  $x'$ . The derivation of this law is particularly simple using our representation. We assume that moist air in the state  $t, x$  contacts a free water surface at its cooling limit temperature  $t'$ . Then, the amount of water  $dW = \sigma (x' - x) dz$  will evaporate in the short time  $dz$ , and the air will lose the energy  $dQ = \alpha (t - t') dz$  to the water. Furthermore, if we consider the equation of the Psychrometer, given before, we have  $Q(x' - x) = C'(t - t')$ , and finally the relation  $dQ = Q dW$  is valid.

The Lewis' law results directly from these four equations. Considering that the absolute values of  $\alpha$  or  $\sigma$  cannot be obtained with great accuracy, and that  $C'$  for typical temperatures of the atmosphere is not much different from the specific thermal capacity of the air, by setting  $C' = 0.25$ , we may write  $\sigma \sim 4 \alpha$ .

---

<sup>8</sup> *W. K. Lewis* "The Evaporation of a Liquid into a Gas", *Mech. Engineering*, 44(7),445-446, 1922.

NT: A correction was later published to this article:

*W. K. Lewis* "The Evaporation of a Liquid into a Gas", *Mech. Engineering*, 55, 567-568,573, 1933.

## References

1. *Mollier*, Ein neues Diagramm für Dampfluftgemische. Zeitschrift des Vereins deutscher Ingenieur, 1923, pp. 869.
2. *Huber*, Zustandsänderungen feuchter Luft in zeichnerischer Darstellung. Zeitschrift des bayerischen Revisions-Vereins, 1924, pp. 79.
3. *Grubenmann*, *ix*-Tafeln feuchter Luft. Berlin, J. Springer, 1926.
4. *Merkel*, Verdunstungskühlung. Mitteilung über Forschungsarbeiten, herausgegeben vom Verein deutscher Ingenieure, Heft 275. Auszug daraus Zeitschrift des Vereins deutscher Ingenieure, 1926, pp. 123.
5. *Merkel*, Der Berieselungsverflüssiger, Zeitschrift für die gesamte Kälteindustrie, 1927, pp. 24.
6. *Merkel*, Der Wärmeübergang an Luftkühlern, Zeitschrift für die gesamte Kälteindustrie, 1927, pp. 117.
7. *Merkel*, Die Berechnung der Verdunstungsvorgänge auf Grund neuerer Forschungen, Sparwirtschaft, Zeitschrift für wirtschaftlichen Betrieb, Wien 1928, pp. 312.
8. *Hirsch*, Die abkühlung feuchter Luft, Gesundheits-Ingenieur, 1926, pp. 376.
9. *Hirsch*, Die Kühlung feuchten Gutes unter besonderer Berücksichtigung des Gewichtsverlustes, Zeitschrift für die gesamte Kälteindustrie, 1927, pp. 97.
10. *Hirsch*, Trockentechnik, Berlin, J. Springer, 1927.
11. *Schlenck*, Das Darren von Malz, Wochenschrift für Brauerei, 1928, Issue 37 a. f.